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A

KEY

TO THE

ABRIDGMENT

OF

DAY'S ALGEBRA;

CONTAINING

MANY EXPLANATIONS, THE ANSWERS TO ALL THE QUESTIONS,

TOGETHER WITH A

STATEMENT AND SOLUTION

OF THE

MORE DIFFICULT PROBLEMS.

BY

JAMES B. THOMSON, A.M.

AUTHOR OF THE ABRIDGMENT OF DAY'S ALGEBRA.

NEW HAVEN:
DURRIE AND PECK.
PHILADELPHIA—HORACE C. PECK.
1850.

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KEY.

SECTION I

ART. 33.

1 See Book.

2 $a-h\times(b+c+d)=37m+\frac{b}{b+h}$.

3 $a+b:\frac{b}{c}::a\times c:12h$.

4. $\frac{a+b+c}{6abc}=4(a+b+c)-d$.

5. $\frac{6}{a+b}=7d-\frac{b}{36}$.

ART. 34.

6. See Book.

7. The product of *a* and *b*, increased by the quotient of three *h* diminished by *c*, divided by the sum of *x* and *y*, is equal to the product of *d* multiplied by the sum of *a*, *b* and *c*, diminished by the quotient of *h*, divided by the sum of 6 added to *b*.

8. *a* increased by 7 times the sum of *h* and *x*, diminished by the quotient of *c* diminished by six *d*, divided by the sum of two *a* increased by four, is equal to the sum of *a* and *h*, multiplied into *b* diminished by *c*.

9. *a* diminished by *b*, is to the product of *a* and *c*, as the quotient of *d* diminished by 4, divided by *m*, is to three times the sum of *h*, *d* and *y*.

10 The quotient of a diminished by h, divided by the sum of 3 and b diminished by c, increased by the quotient of d added to the product of a into b divided by $2m$, is equal to the quotient arising from the product of b and a, multiplied into the sum of d and h, divided by the product of a into m, diminished by the quotient arising from the product of c into d, divided by h increased by the product of d and m

ART. 35

11 and 12. See Book

13. $\frac{4+18}{2}+4\times2\times8-\frac{6\times4\times2\times10}{5\times3}=43.$

14. $4\times8+\frac{(3\times4)+(3\times6)}{6}-\frac{3\times4\times10-6\times2}{3\times2\times6}=34.$

15. 118
16. 9
17. 24.
18. 4.

SECTION II

ADDITION.

ART. 50

1. See Book.
2 $13xy$.
3. $23b+11xy$.
4. $12xy+9abh$.
5. $15cdxy+19gm$
6. See Book.
7. $-6ax$.
8. $-10ab-12my$
9. $-9ach-16bdy$.

ART. 51

10. See Book.
11. $-2b$.
12. $-2bc$.
13. $-7hm$.
14. $3dy+5m$.
15. $8h+3dx$

ART. 53.

16. See Book.
17. $-7xy$.
18. 0.
19. $12abm$.
20. $2axy$.
21. See Book.
22. $aa+aaa+xx+xxx+xxxx$.

ART. 56.

23. See Book.

Examples for Practice.

1. $6ab+7+cd-4m$.
2. $3y-dx-1+hm$.
3. $abm+x+bm-5y+16$.
4. $8am-11+3xy$.
5. $9ahy+16$.
6. $11ad+xy$.
7. $4ab-3ay+x+bx-h$.
8. $by-3ax+3a+3bx$.
9. $6ax+xy$.
10. $44cdf-3xy+6b$.
11. $3bz-17xy+18a+4ax-5bx+63cx$.
12. $8ab-6bc+4cd-7xy+17mn+18fg-2ax$.
13. $8abc+25abd+5xyz$.
14. $4ax+74y+30+3df$
15. $55a+68b$.
16. $7(a+b)$.
17. $2xy(a+b)$.
18. $2ax+5aa+3x+3xxx$.
19. $7y+9yy+5xy-6xx$.
20. $9aaa$.
21. $7yyyy+12xx$.
22. $-12(x-y)-13-6(a+b)$
23. $9a(x+y)-6y+4(a-b)$.
24. $15axy+3bcd$.
25. $17x+17y-9x(a-b)$.
26. $2(x+y)+15abc$.
27. $-6abc+24xy+4mn-25z$
28. $\overline{4a+7b}(x+y)$.

SECTION III

SUBTRACTION.

ART. 60.

1. See Book.
2. $4b$.
3. $8ad$.
4. -12.
5. $-4b$.
6. $-8ad$.
7. $-12b$.

8 $-4b$.
9. $-8ad$.
10. 12.
11. $4b$.
12. $8ad$.
14. $28b$.
15. $20ad$.
16. -44.
17. $-28b$.
18. $-20ad$.
19. See Book.
20. $6aay-17ay$.
21. $16aaxx-20ax$.
22. $6dd+3d-4ddd-10dc-$
$2dddd-4dy$.

ART. 63

23. See Book.
24. $-2h+12bx$.
25. $-4hy+5ah$.
26. $-4dn-6by$.
27. $10abm-7xy$.
28. $3+5ax$.
29. $5ax-8b$.
30. $10ah$.

ART. 65.

31. See Book.
32. $y+2a$.
33. $2ax+bc$.
34 $-3ad+4cd-8bx$

ART. 66.

35. See Book.

Examples for Practice.

1. $2ab+4xy+10dfg$.
2. $-25ax-6ab-7m$.
3. $-3ay-31bx-9bc$.
4. $-16xy+2ab-4d$.
5. $11a+3x+4df+18xyz$.
6. $17bc-42xy+23gh$.
7. $21ax+y+ac-ay-4a+bc$
$-x+yz+dc$.
8. $21x+40xy-13a-42$
$10ab+5bc$.
9. $5xy-29ab$.
10. $-2ax+13ay$.
11. $a+b+c+d-f+g-h-xy$.
12. $13ab+4xy-6ad$.
13. $-(a-b+c+d-f-gh)$.
14. $-(-ab+cdx-df+x+y$
$-ghf+bc-xyz)$.
15. $xx+2bbb$.
16. $5yy+24aaa$.
17. $-10(a+b)+16(x+y)$.
18. $-13(a+b)-52(x-y)$.
19. $a+3aa+7ab$.
20. $-2xx+x-11xxx$.
21. $17+17x$.
22. $13(a-y)-20(a+y)$.
23. $-5ax+5xy-my-52+ay$
$+7df$.
24. $87a$.

SECTION IV.

MULTIPLICATION.

ART. 74.

1. See Book.
2. $24hrxy$.
3. $3dhmy$.
4. $26adghm$.
5 $7bdhx$.
6. $24amxy$.

ART. 76.

7 $3bd+6bxy$.
8 $12dhy+6dmy$
9. $3hhny+my$.
10. $8bhn+12b$.

ART. 78.

11 $6ax+2ad+3hmx+dhm$.
12 $12acy+6bc+4arxy+2brx$.
13 $3ax+3x+4a+4$.
14 $12bd+42d+2b+7$.
15 $6dm+6mrx+6hm+4d+4rx+4h+7dy+7rxy+7hy$.
16 $21r+18br+3adr+28+24b+4ad+14h+12bh+2adh$.

ART. 79.

17 See Book.
18 $bb+2bc+5b+cc+5c+6$.
19. $3ab+3by+3b+2ax+2xy+2x+7a+7y+7$.
20. $6aa+11a+11ad+3dd+13d+4$.
21. $3bb+7bcd+13b+4ccdd+15cd+14$.
22. $6abdx+4adxx+2adhx$.

ART. 80.

23. See Book.
24. $bbbbbbb$.
25. $2x\times3x\times4x\times5x\times6x$; or $720xxxxx$.

ART. 81.

26. See Book.
27. $360abhmy$.
28. $48bdx+24bd$.

ART. 83.

29. See Book.
30. $6ah-3hm+2ax-mx$.
31. $2hy-6dy+8y$.
32. $3ab-6b-21bd-3bx+ah-2h-7dh-hx$.

ART. 84.

33 and 34. See Book.
36. $ab+bb-ax-bx$.
37. $3dmry+hmrx+2mr-3abdy-abhx-2ab$.

38. $3adh+3ad-18h-18$.

ART. 85.

39. See Book.
40. $3ab-12b-6a+24$.
41. $12ad-4ah-28-3addy+adhy+7dy-3adhr+ahhr+7hr$.
42. $8dhy+12dm-4d-4hxy-6mx+2x+6hy+9m-3$.

ART. 86.

43. See Book.
44. $mmmm-yyyy$.
45. $aaa-bbb$.

ART. 87.

46. See Book.
47. $(a+m)\times(h+x)\times(d+y)$
48. $ac+ad+bc+bd$.
49. 0.
50. 0.

Examples for Practice.

1. $4aa+6ab-12a-18bb+8$.
2. $24abmxy-8abx+8abhx$.
3. $240dx(7ah-y)$.
4. $2d(6ab-hd+1)\times(8+4x-1)$.
5. $3adhy+2dhy-4dh+dhh+3ahxy+2hxy-4hx+hhx+3adyy+dyy-4dy+3axyy+xyy-4xy$.
6. $6abhx-4bhh+bdh+6ahx-4hh+dh+6abx-4bh+bd+6ax-4h+d$.
7. $(7ady-d+dh-7axy+x-hx)\times-(r+3-4m)$.
8. $aaa+3aab+3abb+bbb$.
9. $xxx+xxy-xyy-yyy$.
10. $aaccxx+bbccxx+aaddxx+bbddxx+aaccyy+bbccyy+aaddyy+bbddyy$.
11. $aabc-adef+ax-7a+ay+abbc-bdef+bx-7b+by$
12. $aaxy-aayy+10aa-12xy+12yy-120$.
13. $216ab(x+y)$.
14. $3mxyz(a+b+c+d)$.
15. $5ac-5bc-5cc+5ad-5bd+5dd$.
16. $xxx-yyy$.
17. $aaaaaa-bbbbbb$.
18. $aaa+xxx$.

19. $yyyy-aaaa$.
20. $45aa-80bbbb$
21. $\overline{12aa+12ab}(x+y)$.
22. $21xy-18a+2-7c-21xxy+18ax-2x+7cx$.
23. $-2abxxyz+bxyyz-4axxyz+2xyyz$.
24. $50x+12abx-50y-12aby-25mx-6abmx+25my+6abmy$.
25. $aaaa+4aaab+6aabb+4abbb+bbbb$.

SECTION V.

DIVISION.

ART 92.

1. See Book.
2. h.
3. x.
4. y.
5. hx.
6. acd.
7. by.

ART. 94.

8. See Book.
9. bx.
10. $addx$.
11. amy.
12. ahx.
13. y.

ART. 95.

14. See Book.
15. a.
16. $c+d$.
17. $(b+y)\times x$.

ART. 96.

18. See Book.
19. $4y$.
20. $25r$.
21. $2xy$.
22. drx.
23. $20h$.

ART. 97.

24 and 25. See Book.

26. $am\times(h+x+y)$.
27. $4a\times(d+2h+3m+y)$.

28 and 29. See Book.

30. $b+c$.
31. $dh+dy$.

32 $ah+y$

33. $r+h+y$

34. $2b+4c$.

35. $5ry+8$.

36. $3hx+2$.

37. $5m+2x$

Art. 98.

38. a.

39. am.

40. $4a$.

41. ah.

Art. 99.

42 and 43. See Book.

44. a.

45. aa.

Art. 100.

46, 7, 8 and 9. See Book.

50. $-bx$.

51. $-4+5y$.

52. $x-2y$.

53. $-3m\times dh=-3dhm$.

54 and 55. See Book.

Art. 102.

56. $\frac{b}{c}$.

57. $\frac{hx}{y}$.

58. $\frac{hm-3y}{b}$.

59. $\frac{a+x}{y}$.

60. $\frac{am}{xy}$.

Art. 103.

61. See Book.

62. $dy+r-\frac{hd}{x}$.

63. $2h+d+\frac{x}{a}$.

64. $-m-\frac{3y}{b}$.

65. $y+\frac{dh}{2m}$.

Art. 104.

66. See Book.

67, 8 and 9 are each 1.

70. $a+1$.

71. $b-1$.

72. $xy-1+2d$.

73. $ab+1-2m$.

Art. 105.

1. See Book.

Art. 106

2. See Book.

3. $x-y$.

4. $a-b$.

5. $b+c$.

6. $aa-ax+xx$.

7. $3aax-ax+x$.

8. $1-x$.

9. $c+d+\frac{x}{a+b}$.

10. $a+b+\frac{y}{d-h}$.

Examples for Practice.

1. $2ay+ax-3bm+4$
2. $4a-3+2y+1-5adx+\frac{m}{4}$.
3. x
4. a.
5. $-x+\frac{ry}{a}-d+\frac{4my}{a}+\frac{6}{a}-1$.
6. $-\frac{a}{d}-\frac{3}{d}+\frac{x}{d}-\frac{a}{dy}+\frac{1}{my}$.
7. $\frac{ard-6a+2r-hd+6}{2ard}$; or $\frac{1}{2}-\frac{3}{rd}+\frac{1}{ad}-\frac{h}{2ar}+\frac{3}{ard}$.
8. $\frac{3}{2y}-\frac{2}{axy}+\frac{1}{2a}+\frac{1}{axy}-\frac{3h}{2ax}$.
9. $4cx-3xy+6dx-9gh$.
10. $3by+6cdx+2a-5aab$.
11. $4z-2dh+8m$.
12. $a-12b+\frac{14}{x}-24c+10a$.
13. $-10ab+\quad x+y\ -18-3(a+b)-12c$
14. x.
15. 2.
16. $d-x(a+b)+42xy$.
17. $-3+\frac{5h}{m}-\frac{10}{am}+\frac{6cd}{am}-\frac{17}{2m}$.
18. $\frac{1}{6}+\frac{1}{yz}+\frac{1}{3xy}-\frac{1}{6xyz}+\frac{1}{3}(a+b)$.
19. $\frac{1}{b}+\frac{2}{a}+\frac{1}{c}+\frac{5}{3abc}+\frac{abc}{3}$.
20. $9axy+8ax-10bcm+12a$.
21. $-4x+6-2a-21+5ax+\frac{a}{4}$

22. b.

23. $x+y$

24. x.

25. $\frac{4}{a}(ab+1)-\frac{12}{a}(ab+1)+\frac{10}{a}(ab+1)=\frac{2}{a}(ab+1)=2b+\frac{2}{a}$

26. $2x-b+c+\frac{h}{3a+y}$.

27. $aa+2a-4+\frac{10}{b-3}$.

28. $b+2c$.

29. $4aa+2ab+bb$

30. $xx-2ax+aa$

31 $2yy-3y+2$.

32. $xxxxx+xxxx+xxx+xx+x+1$.

33. $2xx-3x+1$ and -2 Rem.

SECTION VI

FRACTIONS.

Art. 117.

1. See Book.

2. $\frac{3m}{4y}$.

3. $\frac{1}{r}$.

4. $\frac{1}{m}$.

5 $\frac{a}{b}$.

6. See Book.

7 $\frac{r+1}{h-1}$.

Art. 118.

8. See Book.

9 $\frac{dgry}{3gmy}, \frac{6hmy}{3gmy}, \frac{18cgm}{3gmy}$.

10. $\frac{2dx+2hx}{3dx+3hx}, \frac{3ad+3ah}{3dx+3hx}, \frac{3rx+3x}{3dx+3hx}$.

11. $\frac{a-b}{aa-bb}$, $\frac{a+b}{aa-bb}$

12. See Book.

13. $\frac{amy}{my}$, $\frac{bmy}{my}$, $\frac{hy}{my}$, $\frac{dm}{my}$.

14. $\frac{adf}{bdf}$, $\frac{bcf}{bdf}$, $\frac{bde}{bdf}$.

15. $\frac{30bx}{10ab}$, $\frac{2ay}{10ab}$, $\frac{5ab}{10ab}$.

16. $\frac{2by}{2y}$, $\frac{2x}{2y}$, $\frac{cy}{2y}$.

17. $\frac{3xyz}{3azy}$, $\frac{3aby}{3azy}$, $\frac{9acz}{3azy}$, $\frac{ayz}{3azy}$.

18. $\frac{60cx}{20ac}$, $\frac{4ab}{20ac}$, $\frac{4acx}{20ac}$

19. $\frac{14ay}{14by}$, $\frac{10by}{14by}$, $\frac{112ab}{14by}$, $\frac{7by}{14by}$.

20. $\frac{16aac}{4acx}$, $\frac{68acx}{4acx}$, $\frac{4axy}{4acx}$, $\frac{4acxx}{4acx}$, $\frac{ccx}{4acx}$.

ART. 119.

21. See Book.

22. $m-1+dy-\frac{hr}{a}$.

ART. 120.

23. See Book.

24. $\frac{ac-b}{c}$.

25. $\frac{cx-a}{c}\ \frac{b}{}$.

26. $\frac{abx-a+c}{x}$.

27. $\frac{adx+a-b}{d}$.

28. $\frac{hm+dh-dm-dl-r}{h-d}$

29. $\frac{bd-by-c}{d-y}$.

ART. 121.

30. See Book.

31. $\frac{8b+8h}{30a-15m}$.

32. $\frac{1}{168-21a}$.

Examples for Practice.

1. 4.
2. $abcdf$.
3. $4b$.
4. $4y$.
5. $2x$.
6. $\frac{3axy}{4}$.
7. $\frac{bx}{2}$.
8. $3c+6x$.
9. $-2y+4a$.

10. $b+\frac{c}{a}+\frac{dx}{a}+x+m.$

11. $\frac{b}{a}.$

12. $\frac{1}{4y}.$

13. $\frac{x+y}{a+x}.$

14. $\frac{axy-ab}{c+bc}.$

15. $\frac{adx}{dy},\ \frac{cy}{dy}.$

16. $\frac{adgy}{bdgy},\ \frac{bcgy}{bdgy},\ \frac{bdfy}{bdgy},\ \frac{bdgx}{bdgy}$

17. $\frac{ax-b-c}{x}.$

18. $\frac{4am+4bm-x+y}{4m}.$

19. $\frac{2acx}{3bdy}.$

20. $\frac{128aabbccdxx}{896abdx}=\frac{abccx}{7}$

ADDITION OF FRACTIONS.

ART. 122.

1 and 2. See Book.

3. $\frac{3hm-2dr-dd}{3dh}.$

4. $\frac{ay-bd+dm}{dy}.$

5. $\frac{-am+dy}{-my}$, or $\frac{am-dy}{my}$, by changing all the signs in the numerator and denominator. (Art. 116.)

6. $\frac{aa+bb}{aa-bb}.$

7. $\frac{ar-am-dh}{dm-dr}.$

8. $-6.$

9. $\frac{4adx+6bcx+bdm}{bdx}.$

10. $\frac{acxyy+achx+aaxy+2cy}{acy}.$

11. $a+c+xy+\frac{bx+4d+ax}{4x}.$

12. $42+2a-\frac{2b}{c}.$

13. Unite the first and third, reduce all the quantities to the lowest terms, &c.

Ans. $y+\frac{2a+2ab-x+y}{c}.$

14. Reduce the fractions to lowest terms, unite, &c.

Ans. $2a+2x+1$

ARTS. 123 and 4.

15. See Book.

16. $a+\frac{b}{m}$, or $\frac{am+b}{m}.$

17. $3d+\dfrac{h+d}{m-y}$, or $\dfrac{3dm-3dy+h+d}{m-y}$.

18. $5x+\dfrac{a+3b}{c}$, or $\dfrac{5cx+a+3b}{c}$.

SUBTRACTION OF FRACTIONS

Art. 125.

1. See Book.

2. $\dfrac{ad+dy-hr}{dr}$.

3. $\dfrac{ay-dm+bm}{my}$.

4. $\dfrac{17d-9a}{12}$.

5. $\dfrac{by-dy+bm}{my}$.

6. $\dfrac{am+m-dd+d}{dm}$.

7. $\dfrac{3b-4a}{ab}$.

Arts. 126 and 7.

8–10. See Book.

11. $\dfrac{acd+bd+hc}{cd}$.

12. $\dfrac{d+2b-2c}{d}$.

13. $\dfrac{-12a+b-5d+24h}{6}$

14. $\dfrac{ac-cx-bd-by}{bc}$.

15. $\dfrac{ay+by-cx+dx}{xy}$.

16. $\dfrac{ad+ay-bc+cx}{bd+by-dx-xy}$.

17. $\dfrac{2ay-2x-3dy}{2y}$.

18. $x+y-\dfrac{a-b}{c}=\dfrac{cx+cy-a+b}{c}$

19. $\dfrac{xx-yy-10a+10b}{10(x+y)}$.

20. Incorporate the intege with the fractions, reduc to a common denom., &c

Ans $c-$

MULTIPLICATION OF FRACTIONS.

Art. 130.

1. See Book.

2. $\dfrac{4ah+4dh}{my-2y}$.

3. $\dfrac{4h(a+m)}{3(a-n)}$.

4. $\dfrac{4a+4h-am-mh}{3c+3y+cd+dy}$.

5. $\frac{3}{8a+24r}$

6. $\frac{acm}{bdy}$.

7 $\frac{2abh-2abd}{cmry-cmy}$.

8. $\frac{3d+bd}{hnr+2hn}$.

9. $\frac{3aad-18ad}{7dhy+7hy}$.

Art. 131.

10. See Book.

11. $\frac{ah}{6}$.

12. $\frac{am+dm}{ah}$.

13. $\frac{3amr+3dr}{5am}$.

Art. 132.

14 and 15. See Book.

Art. 133.

16. See Book.

17. $3m$.

18. $h+3d$.

19. $\frac{a}{b}$

20. $\frac{h}{4}$.

Examples for Practice.

1. $\frac{4aaa}{21}$.

2. $\frac{ax+xx}{aa-ax}$

3. $\frac{5xx}{2b}$.

4. $\frac{3x+y}{3a+4c}$.

5. $\frac{a+b}{4+5y}$.

6. $\frac{4a}{b}$

7. 1.

8. $\frac{y}{x}$.

9. $\frac{3c}{4d}$.

10. $\frac{aa-bb}{12}$.

11. $2abc$.

12. $\frac{9by}{4}$.

DIVISION OF FRACTIONS.

Art. 134.

1 and 2. See Book

3. $\frac{xy+dy}{5dr}$.

4. $\frac{ad}{rx}$.

5. $\frac{4dy}{h}$.

6. $\frac{abx+x}{3aby-3y}$.

7. $\frac{ah-amy+h-my}{12}$.

ART. 135.

8 and 9. See Book.

10. $\frac{1}{8}$.

ART. 136.

11. See Book.

12. $\frac{2xy}{a+b}$.

13 $\frac{4abd+4cdx}{am}$.

14. $\frac{9ac-3x}{a}$.

Examples for Practice.

1. $\frac{c}{x-y}$.

2. $\frac{2x+3b}{10-y}$.

3. $\frac{3x+11}{3ax}$.

4. $\frac{a+1-x}{2cdd}$.

5. $\frac{ab+bb}{ax}$.

6. $\frac{4x+2mx}{3aby}$.

7. $\frac{aa-bb}{12}$.

8. $\frac{x+y}{b}$.

9. 1.

10. $3cx$.

11. $\frac{4cxy}{ab}$.

12. $\frac{27mx}{x-y}$.

13. $\frac{6m(a+x)}{a(a-y)}$.

14. $\frac{4ax+6c+2d}{2xx+xy+bx}$.

SECTION VII.

SIMPLE EQUATIONS.

ARTS. 142–147.

1, 2, 3 and 4. See Book.

5. $x=4+b$.

6. $y=2ab-2hm-a$.

7. $x=b-d-7h-22$.

8. $x=9+8bh$.

9. $x=15$
10. $x=11$
11. $x=20$.
12. $x=31$.

ART. 149.

13. See Book.
14. $x=94$.
15. $x=ah+bh-ad-bd$.
16. $x=4$.

ART. 151.

17. See Book.

ART. 152.

18. $dgmx=abgm+adem-adgh$.

Obser. As the pupil is supposed to be unacquainted with the mode of removing co-efficients, he cannot yet be expected to find the value of x.

19. $15x=134$.

ART. 153.

20. $ar-dr=crx-3bx+2hmx+6nx$.
21. $x=72$.
22. By removing the first denominator, $4x=3+3x+4$.
By transposition, $x=7$. (Art. 152, note.)
23. $x=10$.
24. $37x-36x=70$
$x=70$.

ART. 154

25. See Book.
26. $x=\frac{ah-cd+4bch}{2ch}$

ART. 155.

27. See Book.
28. $x=\frac{h-4}{a+1}$.
29. $x=\frac{ah+dh-4b}{4h-4}$.

ART. 156.

30 and 31. See Book.
32. Dividing by $(a+b)$.
$x-1=d$, and $x=d+1$.

ART. 157.

33. See Book.
34. $ay+by=ch-cm$.

$$y=\frac{ch-cm}{a+b}.$$

ART. 158.

35. See Book.
36. $a+b:c::h-m:y$.
37. $x=2$.
38. $x=3$.
39. $x=12$.
40. $x=2$.

ART. 159.

41. See Book.
42. 104.

43. $x=ad-\frac{ab}{c}=3275$.

44. See Book.

45. $x=\frac{abc(l-m-n)-d(l-m-n)}{c}$.

46. $x=\frac{cdm(b+c+d)+am}{b+c+d}$.

Examples for Practice.

1. $x=8$.

2. $x=\frac{abcd-abch}{bc+ab-ac}$.

3. $x=12$.

4. $x=\frac{93}{4}$, or $23\frac{1}{4}$.

5. $x=\frac{1200}{47}$, or $25\frac{25}{47}$.

6. $x=\frac{1-a}{9}$.

7. $x=-\frac{37}{10}$, or $-3\frac{7}{10}$.

8. $x=\frac{4}{5}$.

9. $x=6$.

10. $x=1\frac{1}{5}$.

11. $x=9$.

12. $x=7$.

13. $x=4$.

14. When the denominator 16, is removed, the numerators of the other fractions become multiples of their denominators. (Art. 152.) $x=9$.

15. By removing the denominator 12 first, the others will disappear.
$x=7$.

16. $x=1$.

17. $x=4$.

18. $x=6$.

19. $x=4$.

20. $x=2$.

SOLUTION OF PROBLEMS.

Art. 161.

1 and 2. See Book.

3. Let $12x$=the estate ;*
Then the shares are, $6x-1000$; $4x-800$; and $3x-600$;
By the conditions, $13x-2400=12x$;
$x=2400$, and $12x=28800$. Ans.

---

* *Obser.* When the conditions of the problem contain fractional expressions, as ½, ⅓, ¼, &c., we can avoid these fractions, and oftentimes

4. 12 the less; 36 the greater.
5 and 6. See Book.
7. $x=0$.
8. $48=$the number.
9. $120x=$the estate.

Then the several shares will be $30x+200$; $24x+340$; $20x+300$; and $15x+400$;

By the conditions, $89x+1240=120x$.

$x=40$; and $120x=4800$. Ans.

Or, let $x=$estate, &c.

10. 450.
11. 200 the less; 240 the greater.
12. Let $x=$C's share;

By the conditions, $400+\frac{x}{3}=x$.

C's$=600$; B's$=400$; A's$=200$; \$1200. Ans.

13. 8.
14. 32½ miles.
15. 480.
16. 1125 and 875.
17. Let $60x=$sum;

Then $20x+15x+12x=94$;

$x=2$; and $60x=\$120$. Ans.

18. Let $x=$number of hours they travel, &c.

Then $x=20$. A goes 200 miles; B 160.

19. 48 years.
20. 1920.
21. 35 feet.

---

greatly abridge the operation by representing the quantity sought by such a number of x's as can be divided by each of the denominators without a remainder. This number can be easily found by taking the least common multiple of all the denominators

22. 100.

23. Let $40x=$ the number

Then $30x+4x+5x+20=40x$

$x=20$; and $40x=800$. Ans.

24 Let $x=$the number of gallons;

Then $\frac{80}{x-7}=$price per gallon;

And $\frac{80}{x-7}\times x=94$.

$x=47$. Ans.

25. Let $35x=$the annual income of each;

Then $5x\times 10=$A's debt at the end of 10 years;

And $7x\times 10=$B's savings at the end of 10 years

$70x-50x=160$.

$x=8$; and $35x=280$ dollars. Ans.

26. 84 years.

27. 84.

28. Let $10x=$his income;

then $10x-2x-100=5x+35$.

$x=45$; and $10x=450$ dollars. Ans.

29. Let $42x+48=$the number of pounds of powder;[*]

$$\left.\begin{aligned}\text{then } 28x+42&=\text{nitre},\\ 7x+3\tfrac{1}{2}&=\text{sulphur},\\ 4x+4&=\text{charcoal},\end{aligned}\right\}\text{by the conditions.}$$

$39x+49\frac{1}{2}=42x+48$.

$x=\frac{1}{2}$; and $42x+48=69$ pounds. Ans.

30. Brandy 29 gallons, wine 44, and water 73.

31. A $317, B $951, C $1268, D $2219.

32. 17, 14, 27, 8 and 33.

---

[*] *Obser.* Sometimes the solution of a problem can be shortened by taking a certain number of x's added to a certain number of units for the quantity required. See Prob. 49.

33. Let x=the share of the fourth son.

Ans. 153 shillings.

34. 147.

35. 20 days.

36. Let x=C's age.

Ans. C, 14; B, 42; A, 84 years.

37 Let x=number of shillings per yard.

then $\frac{100}{x}+10:\frac{130}{x}+10::5:6.$

$x=5$ shillings. 20 and 26 yards. Ans.

38. Let $3x+8$=the original stock of each;

Then $5x-40=(2x+32)\times 2$ by the conditions;

$x=104$; and $3x+8=$£320. Ans.

39. 12.

40. $40 the harness, $80 the horse, and $240 the chaise.

41. 126 gallons.

42. 10, 14, 18, 22, 26 and 30 years.

43. 19 and 30.

44. 16 and 24.

45. 12 gallons, and 36 gallons.

46. 26 and 42.

47. The position of the places may be represented thus:

Let $12x$=the distance from A to B;

Then will $18x$=the distance from C to D;

And $4x$=the distance from B to C;

By the conditions, $34x=34$; and $x=1$.

A to B=12 miles; B to C=4 miles; and C to D=18 miles. } Ans.

48. Let $2x$, $3x$ and $4x$=the numbers; then

$9x=36$; and $x=4$;

8, 12 and 16. Ans.

49. Let $27x+200$=the stock;

Then will $36x+200$=sum at the close of the 1st year.

And $48x+200=$ sum at close of 2d year.
And $64x+200=$ " " 3d "
By the conditions, $64x+200=54x+400$;
£740. Ans.

50. 24000. Ans.

SECTION VIII.

INVOLUTION.

Art. 166.

1, 2 and 3. See Book.
4. $27x^3$.
5. $256y^4$.
6. $128a^7$.

Art. 167.

7. See Book.
8. $b^3m^3x^3$.
9. $a^nd^ny^n$.
10. $d^4h^4y^4$.
11. $(4b)^3$, or $64b^3$.
12. $(6ad)^n$, or $6^na^nd^n$.
13. $(3m\times 2y)^3$, or $216m^3y^3$.

Art. 170.

14. See Book.
15. $a^{3\cdot 4}b^{2\cdot 4}=a^{12}b^8$.
16. $64a^6x^3$.
17. $1296a^{12}x^8d^4$.
18. $(a+b)^{10}$.
19. $(a+b)^{2n}$.
20. $(x-y)^{mn}$.
21. $(x+y)^{2n}$.
22. a^6b^6.
23. $a^9b^6h^{12}$.

Art. 171.

24. See Book.
25. $\frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^n}$
26. $\frac{8x^3r^6}{27y^3}$.
27. $\frac{x^{2n}r^n}{a^ny^{mn}}$.
28. $\frac{a^6(d+m)^2}{(x+1)^6}$.

Art. 172.

29. See Book.
30. $a^2-2ab+b^2$.
31. a^3+3a^2+3a+1.
32. $a^2+2ab+2ah+b^2+2bh$
$+h^2$.

33. $a^3+6a^2d+9a^2+27a+12ad^2+36ad+8d^3+36d^2+54d+27$.

34. $b^4+8b^3+24b^2+32b+16$.

35. $x^5+5x^4+10x^3+10x^2+5x+1$.

36. $1-6b+15b^2-20b^3+15b^4-6b^5+b^6$.

ART. 173.

37 and 38. See Book.

39. $4a^2+4ab+b^2$.

40. h^2+2h+1.

41. $a^2b^2+2abcd+c^2d^2$.

42. $36y^2+36y+9$.

43. $9d^2-6dh+h^2$.

44. a^2-2a+1.

ART. 174.

45. See Book.

46. $(bc+8+x)^n$.

ARTS. 175 and 6.

47 and 48. See Book.

49. $a^2+2ab+2ah+b^2+2bh+h^2$.

ART. 181.

1. See Book.

2. $d^5+5d^4h+10d^3h^2+10d^2h^3+5dh^4+h^5$.

3, 4. See Book.

5. $729x^6+2916x^5y+4860x^4y^2+4320x^3y^3+2160x^2y^4+576xy^5+64y^6$.

6. $a^2-2ab+b^2$.

7. $a^3-3a^2b+3ab^2-b^3$.

8. $a^4-4a^3b+6a^2b^2-4ab^3+b^4$.

9. $x^6-6x^5y+15x^4y^2-20x^3y^3+15x^2y^4-6xy^5+y^6$.

10. $a^n-Aa^{n-1}b+Ba^{n-2}b^2-Ca^{n-3}b^3$, &c.

ART. 182.

11. See Book.

12. $a^4-4a^3+6a^2-4a+1$.

13. $1-6y+15y^2-20y^3+15y^4-6y^5+y^6$.

14. $1+Ax+Bx^2+Cx^3+Dx^4$, &c.

ART. 183.

15. See Book.

ART. 183.a.

16. See Book.

17. $\dfrac{9a^2+12a+4}{9}$.

18. $x^2-bx+\dfrac{b^2}{4}$; or $\dfrac{4x^2-4bx+b^2}{4}$.

19. $\dfrac{9m^2x^2y^2-6bmxy+b^2}{m^2}$.

20. $\dfrac{196a^2b^2c^2-168abc+36}{49}$.

Examples for Practice.

1. $x^3+3x^2y+3yx^2+y^3$.
2. $a^4+4a^3b+6a^2b^2+4ab^3+b^4$.
3. $a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6$.
4. $x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$.
5. $x^8-8x^7y+28x^6y^2-56x^5y^3+70x^4y^4-56x^3y^5+28x^2y^6-8xy^7+y^8$.
6. $m^7+7m^6n+21m^5n^2+35m^4n^3+35m^3n^4+21m^2n^5+7mn^6+n^7$.
7. $a^9+9a^8b+36a^7b^2+84a^6b^3+126a^5b^4+126a^4b^5+84a^3b^6+36a^2b^7+9ab^8+b^9$.
8. $x^{10}+10x^9y+45x^8y^2+120x^7y^3+210x^6y^4+252x^5y^5+210x^4y^6+120x^3y^7+45x^2y^8+10xy^9+y^{10}$.
9. $x^{13}-13x^{12}y+78x^{11}y^2-286x^{10}y^3+715x^9y^4-1287x^8y^5+1716x^7y^6-1716x^6y^7+1287x^5y^8-715x^4y^9+286x^3y^{10}-78x^2y^{11}+13xy^{12}-y^{13}$.
10. $a^7-7a^6b+21a^5b^2-35a^4b^3+35a^3b^4-21a^2b^5+7ab^6-b^7$.
11. $a^8+8a^7b+28a^6b^2+56a^5b^3+70a^4b^4+56a^3b^5+28a^2b^6+8ab^7+b^8$.
12. $32+80x+80x^2+40x^3+10x^4+x^5$.
13. $a^3+3a^2c+3ac^2+c^3-3a^2bx-6abcx-3c^2bx+3ab^2x^2+3cb^2x^2-b^3x^3$.

14. Substitute x for $3bc$.

Ans. $a^3+9a^2bc+27ab^2c^2+27b^3c^3$

15. $16a^4b^4-32a^3b^3x+24a^2b^2x^2-8abx^3+x^4$.

16. Substitute x for $4ab$; and y for $5c^2$, &c.

Ans. $16a^2b^2+40abc^2+25c^4$.

17. $27x^3-162x^2y+324xy^2-216y^3$.

18. $125a^3+225a^2d+135ad^2+27d^3$.

ADDITION OF POWERS.

Art. 185.

1, 2 and 3. See Book.

4. $-5x^6y^5$.

5. $9b^m$.

6. $-4a^4y^n$.

7. a^3h^6.

8. $7(a+y)^n$.

Art. 186.

9 and 10. See Book.

11. $18x(a-b)^3$.

12. $5(x+y)^4+15a^2$.

13. $a^3b^2+x^6y^4+a^2b^3-x^2y^4$
$+a^4b^6$.

14. $11a^2bc^3$.

15. $15a^3+10bc^2$.

16. $4(xy-cm)^6$.

SUBTRACTION OF POWERS.

Art. 187.

1. See Book.

2. $-7b^n$.

3. $-h^2b^8$.

4. 0.

5. $3(a-h)^6$.

6. $5(a+b)^4$.

7. $5a^2x^3+9xy^2$.

8 $2a^3(b^2-8)^3$.

9. $7a^2b^3+x^3y^4-a^3b^6+x^2y^3$.

10. $(x^3+y^4)^3-3(a^2-b^3)^5+$
$3(a^2-b^3)^3$.

11. $x(a-b)^3$.

12. $\frac{1}{4}(x+y)^2$.

MULTIPLICATION OF POWERS.

Art. 188.

1. See Book.

2 $a^4h^2b^{-n}$.

3. $-6a^6xy^2$.

4, $bdy^4h^3x^{-n}$

5. $a^2a^3b^3b^2y^2y$; or $a^5b^5y^3$.

Art. 189.

6 $8a^{2n}$.
7 $6x^{7}$.
8. $b^{6}y^{4}$.
9. $a^{5}b^{5}y^{3}$.
10. $(b+h-y)^{n+1}$.
11. $x^{4}-y^{4}$.
12. $8x^{4}y+2x^{3}y-2x^{2}-3x^{2}y$
$+x$.
13. $2x^{5}+x^{4}+3x^{3}-9x^{2}-4x$
-5.

Art. 190.

14. See Book.
15. y^{n-m-4}.
16. a^{-13}.
17. $-a^{-4}$.
18. $-a^{m+n}$
19. $-y^{-n-3}$.
20. See Book.

Art. 191.

21. $a^{2}-y^{2}$.
22. $a^{4}-y^{4}$.
23. $a^{8}-y^{8}$.
24. $a^{8}-a^{2}$.
25. $6a^{2}(x^{2}-y^{3})^{7}$.
26. $\frac{1}{4}(a^{2}+b^{3})^{5}$.
27. $a^{6}-b^{4}$.
28. $x^{4}+2x^{3}y+2x^{2}y^{2}+2xy^{3}$
$+y^{4}$.
29. $a^{5}+32b^{5}$.
30. $a^{4}+a^{2}b-8a^{2}-8b$.

DIVISION OF POWERS.

Art. 192.

1. $-3y^{4}$.
2. $6x^{n}$.
3. $b+3y^{4}$.
4. d.
5. See Book.

Art. 193.

6. See Book.
7. a^{n}.
8. $x^{0}=1$. (Art. 163, 4.)
9. y^{m}.
10. b^{3}.
11. $2a^{n}$.
12. a^{n+1}.
13. $4(b+y)^{n-3}$.

Art. 194.

14. See Book.
15. $-x^{-2}$.
16. h^{3}.
17. $3a^{n+3}$.
18. $a^{2}b$.
19. b^{-2}.
20. a^{-3}.
21. $(a^{3}+y^{3})^{m\ n}$
22. $(b+x)^{n-1}$.

Compound divisors with indices.

23. a^2-ax+x^2.
24. $a^2+2ax+2x^2$.
25. $x^5+x^4+x^3+x^2+x+1$.
26. $a+b$.
27. $b^6+2b^4c^2+4b^2c^4+8c^6$.
28. $a^2-x+\dfrac{x^4}{a^4-x^3}$.
29. $a^2+2ab+b^2$.
30. $4x^2+2xy+y^2$.
31. $x^2-2ax+a^2$
32. $2y^2-3y+2$.
33. $x^5-x^4+x^3-x^2+x-1$.
34. $2x^2-3x+1$, and -2 rem.
35. $a^3-2a^2b+4ab+4ab^2-8b^2-8b^3$, and $16b^3+19b^4$ rem.
36. x^2+ax-a^2.

Art. 194.*a*.

37. See Book.
38. y^2+ay+a^2.
39. $y^3+ay^2+a^2y+a^3$.
40. $y^4+ay^3+a^2y^2+a^3y+a^4$.

GREATEST COMMON MEASURE.

Art. 196.

1. See Book.

2. $x^3-b^2x\div x=x^2-b^2$.

Then $x^2-b^2)x^2+2bx+b^2(1$

$$\frac{x^2-b^2}{}$$

Dividing this rem. $2bx+2b^2$ by $2b$, (Art. 196.)

We have, $x+b$,

And $x^2-b^2\div x+b=x-b$.

Hence, $x+b$. Ans.

3. $a^2c+a^2x\div a^2=c+x$.

Then, $cx+x^2\div c+x=x$.

$c+x$. Ans.

4. Divide one of the quantities by 2, and the other by 3,

And each will become, x^3-8x-3.

Hence, x^3-8x-3. Ans.

5. $a^5-b^2a^3\div a^3=a^2-b^2$;

Then $a^4-b^4\div a^2-b^2=a^2+b^2$.

Ans. a^2-b^2.

6. Ans. $x+1$.

7. Dividing, &c. $x^3-a^3)x^4-a^4\,(x$

$$x^4-a^3x$$

Divide the remainder by a^3, a^3x-a^4

$x-a$. Ans.

8. Divide the 1st by the 2d; then divide the remainder by $2b$, &c.

$a-2b$. Ans.

9. Divide the 1st by the 2d, and the remainder by $2x^3$, &c

a^2-x^2. Ans.

10. Divide the 1st by the 2d, and the remainder by $-2ab$, &c.

$a+b$. Ans.

FRACTIONS CONTAINING POWERS.

Art. 197.

1. See Book.

2. $2x$.

3. $\frac{3a+4a^3}{5}$.

4. Divide each term by $2ay$. $\frac{4a^2-6ay+3y^2}{3a+2y}$. Ans.

5. See Book.

6. $\frac{2a^8}{5a^7}$ and $\frac{5a^5}{5a^7}$, or $\frac{2a^3}{5a^2}$ and $\frac{5}{5a^2}$.

7. $\frac{3d}{8x^4}$.

8. $\frac{a^4+ab-a^3b-b^2}{3b^4}$.

9. $\frac{a^5b^2+b^2-a^5-1}{x^3+ax^2}$.

10. $\frac{b^4h^{-3}a^n}{a^{-2}xy^{-3}}$.

11. $\frac{a}{y}$.

12. $\frac{a^4-ax^4}{a^2x^2-1}$.

13. $\frac{by^2-y}{a^3+b^{-4}}$.

14. $\frac{h^4-h}{d^{n+4}+d^n}$.

SECTION IX.

POWERS OF ROOTS.

Art. 206.

1. 4th power of the 3d root.
2. 3d power of the 2d root.
3. 8th power of the 6th root.
4. 7th power of the 8th root.
5. $a^{\frac{4}{3}}$.
6. $d^{\frac{7}{5}}$.

Art. 208.

7. See Book.
8. $a^{0.25}$.
9. $a^{0.4}$.
10. $a^{3.5}$.
11. $a^{1.8}$.
12. $a^{2.75}$.
13. See Book
14. $a^{1.66666}$.
15. $a^{1.57142}$.

EVOLUTION.

Art. 210.

1 and 2. See Book.

3. $(ab)^{\frac{1}{3}}$.
4. $a^{\frac{2}{n}}$.
5. $(2d—x)^{\frac{1}{7}}$.
6. $(a—x)^{\frac{3}{5}}$.
7. $a^{\frac{1}{6}}$.
8. $a^{-\frac{1}{4}}$.
9. $a^{\frac{2}{9}}$.
10. $x^{\frac{m}{n}}$.
11. a^2.
12. x^2.
13. $x^{\frac{n}{2}}$.
14. $d^{\frac{3}{5}}$
15. $a^{\frac{3}{8}}$.

Art. 210.*a.*

16. See Book.
17. $(3y)^{\frac{1}{3}}$.
18. $(abh)^{\frac{1}{6}}$.
19. $2b^{\frac{1}{3}}$.
20. $xy^{\frac{1}{n}}$.

Art. 211.

21. See Book.
22. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

23. $\sqrt{\frac{x}{ay}}$.

ART. 216.

1. See Book.
2. $x-1$.
3. $a+\frac{1}{2}$.
4. $a+\frac{2}{3}$.
5. $a+\frac{b}{2}$.
6. $a+\frac{b}{c}$.

REDUCTION OF RADICALS.

ART. 220.

1. See Book.
2. $(64)^{\frac{1}{3}}$.
3. $(81a^4)^{\frac{1}{4}}$.
4. $(\frac{1}{9}a^2b^2)^{\frac{1}{2}}$.
5. $\sqrt[3]{27\times(a-x)^3}$.
6. $(a^6)^{\frac{1}{3}}$.
7. $(a^6b^8)^{\frac{1}{2}}$.
8. $(a^{mn})^{\frac{1}{n}}$.

ART. 221.

9 and 10. See Book.

11. $(a^{2n})^{\frac{1}{n}}$ and $b^{\frac{1}{n}}$.
12. $(x^m)^{\frac{1}{mn}}$ and $(y^n)^{\frac{1}{mn}}$
13. $8^{\frac{1}{6}}$ and $9^{\frac{1}{6}}$.
14. $\sqrt[3]{(a+b)^6}$ and $\sqrt[3]{(x-y)^2}$.
15. $(a^5)^{\frac{1}{15}}$ and $(b^3)^{\frac{1}{15}}$.
16. $(x^4)^{\frac{1}{6}}$ and $(5^3)^{\frac{1}{6}}=(125)^{\frac{1}{6}}$.

ART. 222.

17 and 18. See Book.

19. $(4^3)^{\frac{1}{6}}$ and $(3^2)^{\frac{1}{6}}$.
20. $(x^{10})^{\frac{1}{5}}$ and $(y^{20})^{\frac{1}{5}}$.
21. $(a^3)^{\frac{1}{6}}$ and $(b^2)^{\frac{1}{6}}$.
22. $(c^{\frac{8}{3}})^{\frac{3}{4}}$ and $(d^{\frac{4}{6}})^{\frac{3}{4}}$.
23. $(a^{\frac{n}{m}})^{\frac{1}{n}}$ and $(b^{\frac{2n}{m}})^{\frac{1}{n}}$.
24. $(a^6)^{\frac{1}{12}}$, $(b^4)^{\frac{1}{12}}$ and $(c^3)^{\frac{1}{12}}$

ART. 223.

25 and 26. See Book.

27. $\sqrt{9\times2}=3\sqrt{2}$.
28. $\sqrt[3]{64b^3}\times\sqrt[3]{c}=4b\sqrt[3]{c}$.
29. $\frac{a}{c}\sqrt[4]{\frac{b}{cd}}$.
30. $a\sqrt[3]{b}$.
31. $\sqrt{a^2\times(a-b)}=a(a-b)^{\frac{1}{2}}$.
32. $\sqrt[3]{27a^6\times2b}=3a^2(2b)^{\frac{1}{3}}$.
33. $\sqrt{49a^2\times2x}=7a(2x)^{\frac{1}{2}}$

34. $\sqrt[3]{a^3\times(1+b^2)}=a(1+b^2)^{\frac{1}{3}}$.

ART. 224.

35 and 36. See Book.

37. $(16a^4b^5)^{\frac{1}{3}}$.

38. $\left(\dfrac{a^2b^2c}{a^2b^2+b^4}\right)^{\frac{1}{2}}$.

39. $\sqrt{4\times2}=\sqrt{8}$.

40. $\sqrt[3]{64b^3\times c}=(64b^3c)^{\frac{1}{3}}$.

Examples for Practice.

1. $\sqrt{150}$.
2. $\sqrt{\dfrac{a}{5}}$.
3. $125^{\frac{1}{6}}$ and $36^{\frac{1}{6}}$.
4. $(a^8)^{\frac{1}{4}}$ and $(a^2)^{\frac{1}{4}}$.
5. $7\sqrt{2}$.
6. $9\sqrt{3}$.
7. $3\sqrt[3]{2}$.
8. $7\sqrt{16\times5}=28\sqrt{5}$.
9. $9\sqrt[3]{27\times3}=27\sqrt[3]{3}$.
10. $x\sqrt{1+a}$.
11. $3a\sqrt{22x}$.
12. $x(x-a^2)^{\frac{1}{2}}$.

ADDITION OF RADICALS.

ART. 225.

1. $3\sqrt[4]{ay}$.
2. $3\sqrt{a}$.
3. $7(x+h)^{\frac{1}{7}}$.
4. $12bh^{\frac{1}{6}}$.
5. $(a+y)\sqrt{b-h}$.

ART. 226.

6. See Book.
7. $4\sqrt{b}+2\sqrt{b}=6\sqrt{b}$.
8. $a\sqrt{x}+b^2\sqrt{x}=(a+b^2)\sqrt{x}$.
9. $(6a+5)\sqrt{y}$.
10. $6\sqrt{2a}$.

11 and 12. See Book.

Examples for Practice.

1. $7\sqrt{3}$.
2. $14\sqrt{2}$.
3. $15\sqrt{5}$.
4. $9\sqrt[3]{5}$.
5. $32\sqrt[3]{2}$.
6. $191\sqrt{3}$.
7. $16\sqrt{b}$.
8. $7a\sqrt{d}$.
9. $11x^2\sqrt{c}$.
10. $7a^2\sqrt[3]{b}$.

SUBTRACTION OF RADICALS.

Art. 229.

1. See Book.

2. $(a+x)^{\frac{1}{n}}$.

3. $8h^{\frac{1}{3}}$.

4. $(a-b)\sqrt[4]{x+y}$.

5. $a^{-\frac{1}{n}}$.

6. $5\sqrt{2}-2\sqrt{2}=3\sqrt{2}$.

7. $(b-y)\sqrt[3]{by}$.

8. $x^{\frac{1}{n}}-x^{\frac{1}{3}}$.

9. $7\sqrt{2}$.

10. $2\sqrt[3]{5}$.

11. $10\sqrt{5}-9\sqrt{5}=\sqrt{5}$.

12. $(4a^2-2a)\sqrt{5x}$.

MULTIPLICATION OF RADICALS.

Arts. 230 and 31.

1, 2 and 3. See Book.

4. $(a^2-m^2)^{\frac{1}{2}}$.

5. $(dhxy)^{\frac{1}{2}}$.

6. $(a^3x)^{\frac{1}{2}}$.

7. $\sqrt[n]{(a+y)\times(b+h)}$.

8. $(a^nx^m)^{\frac{1}{mn}}$.

9 and 10. See Book.

11. $(a^4y^4)^{\frac{1}{4}}=ay$.

Art. 233.

12 and 13. See Book.

14. $3y^{\frac{11}{12}}$.

15. $(a+b)^{\frac{3}{4}}$.

6. $(a-y)^{\frac{m+n}{mn}}$.

17. $x^{-\frac{7}{12}}$.

18. $y^{\frac{2}{3}-\frac{2}{4}}=y^{\frac{1}{6}}$.

19. $a^{\frac{1}{n}-\frac{1}{n}}=a^0=1$.

20. $x^{(n-n+\frac{1}{2}-\frac{1}{2})}=x^0=1$.

21. $a^{\frac{7}{3}}$.

22, 23 and 24. See Book.

25. $(a+b)^{\frac{3}{3}}=a+b$.

26. $a^{\frac{5}{5}}=a$.

Art. 234.

27, 28 and 29. See Book.

30. $ay(b^2-x^2)^{\frac{1}{2}}$.

31. $ab\sqrt{hy^3}$.

32. abx.

33. $ab(x^{-\frac{1}{2}}y^{-\frac{1}{2}})=ab(xy)^{-\frac{1}{2}}$.

34. $3xy$.

35 and 36. See Book.

Examples for Practice.

1. $(a^3b^2)^{\frac{1}{6}}$.
2. $30\sqrt{10}$.
3. $6(432)^{\frac{1}{6}}$.
4. $(a^2b^2d^3)^{\frac{1}{6}}$.
5. $\sqrt{\frac{3a^2d}{c}}$.
6. $(ac—ad)\times(a^2x—ax^2)^{\frac{1}{2}}$.
7. $10\sqrt{10}$.
8. $\frac{1}{10}\sqrt{6}$.
9. $15\sqrt{10}$.
10. $27\sqrt{15}$.
11. $8706\frac{1}{6}a^{\frac{11}{12}}$.
12. 4.

DIVISION OF RADICALS.

ART. 237.

1, 2, 3, 4 and 5. See Book.

6. $\sqrt{2a^3}$.
7. $\sqrt{hx}$.
8. $(a^2+x)^{\frac{1}{3}}$.
9. $\left(\frac{a^2h}{x}\right)^{\frac{1}{m}}$
10. $(ay)^{\frac{1}{4}}$.

11 and 12. See Book

ART. 239.

13. $(3a)^{\frac{1}{4}}$.
14. $(ax)^{\frac{1}{3}}$.
15. $a^{\frac{1}{n}}$.
16. $(b+y)^{\frac{1}{n}}$.
17. $(r^2y^3)^{-\frac{2}{7}}$.

18 and 19. See Book.

ART. 240.

20. $4x\sqrt{y}$.
21. $9d\sqrt{b}$.
22. $b(a^2x)^{\frac{1}{n}}$.
23. $2\sqrt{8}=4\sqrt{2}$.
24. $b\sqrt{x}$.
25. See Book.

Examples for Practice.

1. $\frac{2}{3}\left(\frac{b^2}{a^3c}\right)^{\frac{1}{6}}$.
2. $2\sqrt[3]{27}=6$.
3. 15.
4. $12\sqrt{2}$.
5. $(ab)^{\frac{1}{3}}$.
6. $(4a—3x)^{\frac{1}{2}}$.
7. $3\sqrt{23}$.
8. $8\sqrt[3]{4}$.

9. $\frac{3}{2}\sqrt{\frac{5}{2}}$

10. $7^{\frac{1}{6}}$

11. $6\sqrt{3}$

12. $2\sqrt[3]{4}$.

INVOLUTION OF RADICALS.

Art. 242.

1. See Book.

2. $a^{\frac{1}{4}\times 3}=a^{\frac{3}{4}}$

3. a^m.

4. $a^{\frac{5}{2}}y^{\frac{5}{3}}$

5. $a^{\frac{3}{n}}x^{\frac{3}{m}}=(a^m x^n)^{\frac{3}{nm}}$.

6. $a^{\frac{4}{3}}x^{\frac{6}{4}}$.

7. $a^{\frac{3}{3}}=a$.

8 $a^{\frac{n}{n}}=a$.

Art. 243.

9, 10 and 11. See Book.

12. $a^{mn}x^{\frac{n}{m}}$.

13. $a^2(x-y)$

14. $27a^3y$.

Art. 244

15. See Book.

16. $a^3-3a^2\sqrt{b}+3ab-b\sqrt{b}$.

17. $8d^3+12d^2\sqrt{x}+6dx+x\sqrt{x}$.

18. d^2.

19. x^2-2x+1.

20. $a^3+3a^2b+3ab^2+b^3$.

EVOLUTION OF RADICALS.

Art. 245.

1. See Book.

2. $a^{\frac{1}{3}}(xy)^{\frac{1}{6}}=(a^2xy)^{\frac{1}{6}}$.

3. $a^{\frac{1}{n}}(by)^{\frac{1}{6n}}=(a^6by)^{\frac{1}{6n}}$.

4. $a^{\frac{1}{8}}\times b^{\frac{1}{24}}=(a^3b)^{\frac{1}{24}}$.

5. $2d^{\frac{1}{49}}$.

Examples for Practice.

1. $3a^{\frac{1}{2}}$.
2. $(a+b)^{-\frac{1}{2}}$.
3. $(x-y)^{\frac{1}{6n}}$.
4. $-5ax^2$.
5. $\frac{2a^2}{3xy}$.
6. $\frac{2ax^2}{3}$.
7. $x-3b$.
8. $a+\frac{y}{2}$.
9. $(a^6x^{12})^{\frac{1}{6}}$.
10. $(-27y^3)^{\frac{1}{3}}$.
11. $(a^6)^{\frac{1}{3}}$ and $a^{\frac{1}{3}}$.
12. $256^{\frac{1}{12}}$ and $125^{\frac{1}{12}}$.
13. $(a^4$ and $b^3)^{\frac{1}{6}}$.
14. $(16$ and $16)^{\frac{1}{6}}$.
15. $\sqrt{49\times6}=7\sqrt{6}$.
16. $x\sqrt{x-a^2}$.
17. Sum $=6a\sqrt{x}$.
    Diff. $=2a\sqrt{x}$.
18. $4\sqrt[3]{3}+2\sqrt[3]{3}=6\sqrt[3]{3}=$sum.
    And $2\sqrt[3]{3}=$difference
19. $70\sqrt[3]{9}$.
20. 4.
21. $ab(a^2-c)^{\frac{1}{2}}$.
22. $6(a+b)^{\frac{n+m}{mn}}$
23. $2\sqrt{27}=6\sqrt{3}$.
24. $2\sqrt[3]{4}$.
25. $7^{\frac{1}{6}}$.
26. $8\sqrt[3]{4}$.
27. $4913\sqrt{9261}=4913\times21$
    $\sqrt{21}=103173\sqrt{21}$.
28. $27+10\sqrt{2}$.
29. $\frac{1}{36}$.
30. $x\sqrt{x}-3x\sqrt{b}+3b\sqrt{x}$
    $-b\sqrt{b}$; or $x^{\frac{3}{2}}-3x\sqrt{b}+$
    $3b\sqrt{x}-b^{\frac{3}{2}}$

SECTION X.

REDUCTION OF EQUATIONS BY INVOLUTION.

Art. 247.

1 and 2. See Book.

3. $x=63$.
4. $x=\frac{25}{36}+4$.
5. $x=(3+d-a^2)^2$.
6. $x=\frac{361}{100}$, or $3\frac{61}{100}$.
7. $x=20$.
8. $x=12$.

9. $x=4$.

10. $x=\frac{25a}{16}$.

11. $x=\frac{9}{20}$.

12. $x=\frac{1}{1-a}$.

13. $x=4$.

14. $x=\frac{1}{3}a$.

15. $x=a\sqrt{\frac{1}{3}}$.

16. $x=\frac{b^2-4a^2}{4a}$.

17. $x=\frac{2}{3}$.

18. $x=81$.

19. $x=16$.

20. $x=6$.

REDUCTION OF EQUATIONS BY EVOLUTION.

Art. 249.

1 and 2. See Book.

3. $x=\pm\left(\frac{bdh-abd}{b+d}\right)^{\frac{1}{2}}$.

4. $x=\left(\frac{10-a}{d+1}\right)^{\frac{1}{n}}$.

Art. 250.

5. See Book.

6. $x=(h^2-2hd+d^2+a)^{\frac{1}{m}}$.

7. $x=(2a^2+2ab+b^2)^{\frac{1}{2}}$.

8. $(x^2-1)^{\frac{1}{2}}=\frac{8}{(x^2-1)^{\frac{1}{2}}}$.

Removing the denominator,

$x^2-1=8$; and $x=3$.

9. $x=6$.

10. $y=a+b$.

11. $(13+\sqrt{23+y^2})^{\frac{1}{2}}=5$.

Involving each side,

$13+\sqrt{23+y^2}=25$.

Transposing and involving each side again,

$23+y^2=144$.

$y^2=121$.

By evolution, $y=11$.

12. $(3+\sqrt[3]{329+x^2})^2=144$.

$3+\sqrt[3]{329+x^2}=12$.

Transposing and involving each side,

$329+x^2=729$.

Transposing, $x^2=400$.

And $x=20$. **Ans.**

PROBLEMS.

1 and 2. See Book.

3. \$400.

4. 12 miles.

5. 16.

6. 4.

7 Let $10x=$their sum.

Then $7x=$greater.

And $3x=$less.

$30x^2=270$.

$x=3$.

Ans. $\left\{\begin{array}{l}21=\text{greater.}\\ 9=\text{less.}\end{array}\right.$

8. Let $2x=$difference.

Then $2:9::2x:$ greater, i. e. $9x$.

And $9x-2x=7x$, the less.

By the conditions, $81x^2-49x^2=128$.

$x=2$, and $2x=4$, the diff.

Ans. $\left\{\begin{array}{l}\text{18 the greater,}\\ \text{14 the less.}\end{array}\right.$

9. 10 the greater, and 8 the less.

10. Let $2x=$difference.

Then $7+x=$greater:

And $7-x=$less.

$$\frac{63+9x}{7-x}=\frac{112-16x}{7+x}$$

$x=1$, and $2x=2$.

Ans. 8 and 6.

11. Let $4x$ and $5x=$the Nos

Ans. 12 and 15.

12. 126 miles.

13. 24 and 15.

14. Let $x=$longer piece.

Then $36-x=$shorter do

$x^2:(36-x)^2::4:1$

$x^2=4(36-x)^2$.

$x=24$.

Ans. 24 and 12 yards

15. 6 and 4.

16. 12.

17. 12.

AFFECTED QUADRATIC EQUATIONS.

Art. 258.

1. See Book.

2. $x=4b\pm\sqrt{16b^2+h}$.

3. $x=-\dfrac{a}{2}\pm\left(\dfrac{a^2}{4}+b+h\right)^{\frac{1}{2}}$.

4. $x=\dfrac{1}{2}\pm\left(\dfrac{1}{4}+h-d\right)^{\frac{1}{2}}$.

5. $x=-\dfrac{3}{2}\pm\left(\dfrac{9}{4}+d+6\right)^{\frac{1}{2}}$.

6. $x=\frac{ab}{2}\pm\left(\frac{a^2b^2}{4}+ab-cd\right)^{\frac{1}{2}}.$

7. $x=-\frac{a}{2b}\pm\left(\frac{a^2}{4b^2}+h\right)^{\frac{1}{2}}.$

8. $x=\frac{1}{2b}\pm\left(\frac{1}{4b^2}+7h\right)^{\frac{1}{2}}.$

Art. 260.

9. See Book.

10. $x=-\frac{a+b}{2}\pm\sqrt{\left(\frac{a+b}{2}\right)^2+h}.$

11. $x=-\frac{a-1}{2}\pm\sqrt{\left(\frac{a-1}{2}\right)^2+b}.$

Art. 261.

12. See Book.

13. $x=-5\pm\sqrt{81}=4.$

Art. 262.

14. See Book.

15. $x=-\frac{a}{b}\pm\left(\frac{a^2}{b^2}+\frac{ad-ah}{b}\right)^{\frac{1}{2}}.$

Art. 263.

16. $x=\frac{2}{b+d}\pm\sqrt{\left(\frac{2}{b+d}\right)^2+\frac{b-h}{b+d}}.$

17. $x=\frac{1}{a+1}\pm\sqrt{\left(\frac{1}{a+1}\right)^2+\frac{h}{a+1}}.$

Art. 265.

18. $x=\frac{-d\pm\sqrt{4ah+d^2}}{2a}.$

19. $x=3$

20. $x=9$; or 6.

21. $x=1\pm\sqrt{h-d+1}.$

22. $x=8.$

ART 267.

23. See Book.

24. $x=\pm\sqrt[5]{2b\pm\sqrt{(4b^2+a)}}$.

25. $x=(-2\pm\sqrt{h-n+4})^2$.

26. $x=(-4\pm\sqrt{a+b+16})^2$.

ARTS. 269–274.

27–31. See Book.

Examples for Practice.

1. $x=7$, or -4.

2. $x=12$, or $-\frac{3}{4}$.

3. $x=4$, or $-\frac{7}{4}$.

4. $x=4$, or -1.

5. $x=4$, or $2\frac{1}{12}$.

6. $x=12$, or 6.

7. $x=21$, or 5.

8. $x\pm1$, or -28.

9. $x=2$.

10. $x=10$.

11. $x=1\pm\sqrt{1-a^2}$.

12. $x^2+\frac{a}{2}=\pm\sqrt{b+\frac{a^2}{4}}$ Extracting root, &c.

$x=\sqrt{-\frac{a}{2}\pm\left(b+\frac{a^2}{4}\right)^{\frac{1}{2}}}$. Ans.

13. Clearing of fractions and dividing by 2, the equation becomes, $2x^6-x^3=-\frac{4}{32}$.

Completing the square, &c. $4x^3-1=0$.

$x^3=\frac{1}{4}$, and $x=\sqrt[3]{\frac{1}{4}}$. Ans.

14. $x=\frac{1}{8}$.

15. $x=49$.

16. $x=\frac{1}{2}\sqrt{6}$.

17. $x=6$.

18. $x=\sqrt[5]{2}$.

19. $2(1+x-x^2)-\sqrt{1+x-x^2}=-\frac{1}{9}$.

Completing square, (Art. 267,)

$16(1+x-x^2)-8\sqrt{1+x-x^2}+1=\frac{1}{9}$.

$$4\sqrt{1+x-x^2}-1=\frac{1}{3}.$$

$$\sqrt{1+x-x^2}=\frac{1}{3}.$$

$$1+x-x^2=\frac{1}{9}.$$

$x=\frac{1}{2}\pm\frac{1}{6}\sqrt{41}$. Ans.

20. $x=\frac{b}{2}\pm\sqrt{\frac{4a^3-b^3}{12b}}.$

21. $x=4$.

22. $x^{\frac{6}{5}}+x^{\frac{3}{5}}=756.$

Then $4x^{\frac{6}{5}}+4x^{\frac{3}{5}}+1=3025$. (Art. 267.)

$2x^{\frac{3}{5}}+1=\pm55.$

$x^{\frac{3}{5}}=27.$

$x^{\frac{1}{5}}=3$; and $x=243$. Ans.

23. $x=4$.

24. $x=9a$.

25. $x=9$.

26. $\sqrt{x^6}+\sqrt{x^3}=6\sqrt{x}$.
Div. by $\sqrt{x}$, we have $\sqrt{x^4}+\sqrt{x^2}=6$; or, $x^2+x=6$
$x=2$. Ans.

27. $x=2$.

28. $11x^2-59x=-78$.
Completing the square, &c. $x=3$. Ans.

29. $(x-5)^3-3(x-5)^{\frac{3}{2}}=40.$

Then $4(x-5)^3-12(x-5)^{\frac{3}{2}}+9=169$. (Art. 267.)
Extracting the root, $2\sqrt{(x-5)^3}-3=13$.
Transposing and dividing, $\sqrt{(x-5)^3}=8$.
By involution, $(x-5)^3=64$.
By evolution, $x-5=4$; and $x=9$. Ans.

30. Transposing, &c. $x-2=2\sqrt{x+6}$.
$x=10$, or -2. Ans.

PROBLEMS IN QUADRATIC EQUATIONS.

1. See Book.
2. 25 and 20 years.
3. 9 and 13.
4. 6 dollars.
5. 2 and 5.
6. 20 and 32.
7. 40 and 160.
8. 2 and 4.
9. 40 and 16.
10. 15 pieces.
11. 9 and 6 miles.
12. 17 and 11.
13. 120 by A, and 80 by B.
14. 7 and 3.
15. 7.
16. 75 yds. at 80 cts. per yard.
17. A 117 and B 130 miles.
18. 18 of the finer and 20 of the coarser pieces, 20 and 16 shillings the prices per yard.

19. Let $2x=$the number of gallons of Teneriffe.
Then $54x+2x^2-8x=$the cost of both.
By the conditions, $54x+2x^2-8x=20x+540+576$.
Transposing, &c. $x^2+13x=558$.
$x=18$; and $2x=36$, the No. of gallons of Teneriffe; and the Madeira cost 18 shillings per gallon.

20. 6.

21. 16 years.

22. Let $3x=$the number of gallons in smaller cask;
Then $3x+5=$the number of gallons in larger cask;
$x-2=$price per gallon.
$6x^2-7x=68$.
$x=4$.
Ans. 12 and 17 gallons; price $2 per gallon.

23 Let $x=$the number of silver coin;
$24-x=$the number of copper coin;
Then $48x-2x^2=216$.

$x^2-24x=-108$.

Ans. 18 silver coins, 16 copper coins.

24. Let x=the number of oxen;

Then $\frac{80}{x}=\frac{80}{x+4}+1$.

$x=16$. Ans.

25. 10 and 14.
26. 20 and 40.
27. 25 and 121.
28. 2 and 18.
29. $\frac{1}{2}$ and $\frac{5}{6}$.
30. 3 and 18.
31. 7.
32. 16.
33. Let x=the length.

$42-x$=breadth.

By the conditions, $42x^2-x^2=432$.

Then $x=24$ length.
$42-x=18$ breadth. } Ans.

34. A's 15 years, and B's 8 years.
35. 5.
36. 7 rods.
37. Length 15 rods. Breadth 10 rods
38. Let x=number.

By the conditions, $x+\sqrt{x}=156$.
By transposition, $\sqrt{x}=156-x$.
By involution, $x=(156-x)^2$.
Completing square, &c. $x=144$. Ans

39 Let x=length.

Then $\frac{48}{2}-x$=breadth

By conditions, $x-(24-x)\times 35=24x-x^2$.
$x=14$ length, and 10 the breadth.

40. Let $x=$the width of the border.

Then $54x+54x+36x+36x+4x^2=1944$.

Completing square, &c. $x=9$ feet, the width of border

The length of the lot is 72 feet.

And the breadth 54 feet.

And 3888 square feet of land.

41. 177 in rank.
118 in file.

42. 3 inches.

43. Let $x=$A's stock.

Then will $500-x=$B's stock.

$$\left.\begin{array}{l}\dfrac{5x}{3x+1000}\times400=\text{A's gain.}\\[2ex]\dfrac{1000-2x}{3x+1000}\times400=\text{B's gain.}\end{array}\right\}\text{By principles of fellowship.}$$

$x=\$200$, A's stock. And \$300, B's.

44. $x : 39-x : : 100 : x$.

Ans. £30.

45. 75 sheep at 16 shillings a head.

46. A went 117; B 130 miles.

47. A's stock \$480. B's \$420.

48. Length 27 yards; price 12*s.* per yard, N. E. currency.

49. Let $x=$number of gallons first drawn off.

Then $20-x=$number of gallons of wine remaining in the first cask.

$$\overset{\text{Mixt.}}{20} : \overset{\text{Wine.}}{x} : : \overset{\text{Mixt.}}{x} \text{ to } \frac{x^2}{20}=\text{the wine drawn from 2d cask, \&c.}$$

$$20-x+\frac{x^2}{20}=\frac{400-20x+x^2}{20}=\text{the wine in 1st cask after it is filled with the mixture.}$$

Mixt. Wine. Mixt.

$$20 : \frac{400-20x+x^2}{20} :: 6\tfrac{2}{3} \text{ to } \frac{400-20x+x^2}{60} = \text{the wine}$$

in $6\frac{2}{3}$ gallons of mixture.

By conditions, $\frac{400-20x+x^2}{20}-\frac{400-20x+x^2}{60}=10$.

Clearing of fractions, &c., $x^2-20x=-100$.

Completing the square, &c., $x=10$. Ans.

40. Let x=the number of lbs. of pepper for £10.

Then $x+60$=the number of lbs. of ginger for £20.

$\frac{10}{x}$=price per lb. of pepper.

$\frac{20}{x+60}$=price per lb. of ginger.

By the conditions, $\frac{800}{x}+\frac{2000}{x+60}=65$.

$x^2+\frac{220x}{13}=\frac{9600}{13}$

$x=20$. And $\frac{10}{20}=£\frac{1}{2}$, or 10s.

Hence the price per lb. of the pepper, is 10s.

And the price of the ginger, is 5s.

SECTION XI.

TWO UNKNOWN QUANTITIES.

Art. 277.

1. See Book.
2. $x=5$; and $y=6$.
3. $x=10$; and $y=3$
4. $x=6$; and $y=4$.
5. $x=15$; and $y=20$.
6. $x=11$; and $y=9$.
7. $x=20$; and $y=4$.

8 $y=\dfrac{h^2-d}{2h}$.

$x=\dfrac{h^2+d}{2h}$.

9. $y=\dfrac{h-ad}{b-a}$.

Art. 279.

10 and 11. See Book.
12. $x=5$; and $y=2$.
13. $x=2$; and $y=10$.
14. $x=4$; and $y=20$.
15. $x=8$; and $y=12$

16. Let x=the distance the privateer sails.
And y=the distance the ship sails.
1. By the conditions, $x=y+20$.
2. And $x:y::8:7$.
3. $7x=8y$.
4. $y=\dfrac{7x}{8}$.

Substituting value of y in 1, $x=\dfrac{7x}{8}+20$.

Then, $x=160$; and $y=140$.

17. A's age 49; B's 21.

18. 10 and 15.

Art. 281.

19 and 20. See Book.
21. $x=6$; and $y=4$.
22. $x=8$; and $y=6$.
23. $x=12$; and $y=2$.
24. $x=7$; and $y=3$.
25. 11111 the greater; and 9999 the less.
26. For lemons 4 cts.; and oranges 6 cts.
27. 120 the greater; and 100 the less.

Art. 282.

28. 60 feet the lower part.
48 feet the upper part.

29. See Book.

30. 10 the greater; and 2 the less.

31. 3 the greater; and 2 the less.

32. 20 the greater; and 12 the less.

33. 32 the greater; and 20 the less.

34. Let $x=$the tens, and $y=$the units.

Then $x+y=8$.

Since x stands in ten's place,

$10x+y=$the number.

By conditions, $10x+y+36=10y+x$.

Then $x=2$, and $y=6$.

26 is the number.

Obser. This and other similar problems may be solved by using one unknown quantity.

35. Let $x=$the tens.

And $y=$the units.

Then, $x+y=9$.

$10x+y-27=10y+x$.

$x=6$; and $y=3$.

Ans. 63, the sum of their ages.

36. Let $x=$gallons of brandy.

And $y=$gallons of gin.

By the conditions, $\begin{cases} x+6 : y+6 :: 7 : 6. \\ x-6 : y-6 :: 6 : 5. \end{cases}$

Then $6x+36=7y+42$.

$5x-30=6y-36$, &c.

Ans. $\begin{cases} \text{78 gallons of brandy.} \\ \text{66 gallons of gin.} \end{cases}$

THREE UNKNOWN QUANTITIES.

Arts. 284 and 5.

37 and 38. See Book.

39. First, from the three, get two equations.

$2x+y=16$.

$4x+3y=36$.

Then $2x=12$; and $x=6$.

$y=4$; and $z=2$.

Art. 286.

40. $x=\dfrac{a+b-c}{2}$; $y=\dfrac{a+c-b}{2}$; and $z=\dfrac{b+c-a}{2}$.

41 A had \$64; B \$72; and C \$84.

ART. 287.

42. A 46; B 9; and C 7 miles.
43. $x=24$; $y=60$; and $z=120$.
44. $x=30$; $y=20$; and $z=10$.

ART. 288.

45 and 46. See Book.
47. 93. (See solution of problem 34.)
48. 36.
49. 13 and 4.
50. $\frac{4}{21}$.
51. 56 and 33 guineas.
52. Let x, y and z, be three of the parts.

And $90-x-y-z=$ the fourth.

Then 1. $x+2=y-2$.

2. $x+2=2z$.

3. $2z=\frac{90-x-y-z}{2}$, &c.

Find the value of y and z in 1st and 2a, and substitute them in 3d.

Ans. $x=18$, $y=22$, $z=10$, and the 4th$=40$.

53. 50, 65 and 75.
54. 10 and 2.
55. Port, 3 guineas; and sherry, 2.
56. 78 gallons of brandy, and 66 of water.
57. $\frac{4}{5}$.
58. 72 apples, and 60 pears.
59. 13 and 17.

ART. 292.

60. See Book.

SECTION XII.

RATIO.

ART 313

1 *First*, express the ratio of each couplet in the form of a fraction. Thus $\frac{11}{9}$ and $\frac{44}{35}$.

Second, reduce the fractions to a common denominator; and they become $\frac{385}{315}$ and $\frac{396}{315}$, the last of which is greater than the first by $\frac{11}{315}$.

2. The 2d is the greater by $\frac{3}{a}$.

3. By Art. 302, $\frac{65}{13}=5$. Ans.

4. By Art. 302, $7\times 18=126$. Ans.

5. $42ax+6a : 105by-70b$, or $\frac{42ax+6a}{105by-70b}$.

6. By rejecting equal factors from the antecedent and consequent, (Art. 311,) $x^2-y^2 : bh$. Ans.

7. Greater inequality. (Art. 303.)

8. Equality. (Art. 303.)

9. 14 : 15.

10. $3\times 7\times x^3 : 7\times 3\times y^3$.

By Art. 311. $x^3 : y^3$. Ans.

SECTION XIII

ARITHMETICAL PROPORTION AND PROGRESSION.

ART. 330.

1, 2 and 3. See Book.

ARTS. 332–335.

4 and 5. See Book.

6. 440.

7. 5 miles and 1300 yards.

8. 3775.

9. 3.

10. 21

11. 280.

12. \$220.90.

13. \$667.95.

14. 156 strokes.

15. 300 strokes

16. 10201.

17. 730.

18. 301.

19. \$3.40.

Obser. The principal is the first term; the rate, the common difference; and the number of years+1=the number of terms. Hence the last term will be the answer.

20. 3.

21. 19.

22. 13, 20, 27, 34 and 41.

23. 12, 16, 20, 24, 28 and 32.

Art. 336.

24. See Book.

25. Let $x=$ 2d term, and $y=$common difference.

Then $x-y$, x, $x+y=$the series.

Adding the terms, $3x=9$, and $x=3$.

The sum of the cubes, $(x-y)^3+x^3+(x+y)^3=153$.

That is, $3x^3+6xy^2=153$.

$x^3+2xy^2=51$.

Substitution, $27+6y^2=51$.

Then $y=2$; and the series is 1, 3 and 5.

26. Let $x=$ 2d term, and $y=$common difference.

Then $x-y$, x, $x+y=$the series.

By conditions, $x-y+x+x+y=15$

That is, $3x=15$, and $x=5$.

And $2x^2+2y^2=58$.

By substitution, &c., $y=2$.

The series is, 3, 5 and 7.

27 Let $x=$ 2d term, and $y=$common difference.

Then will $x-y$, x, $x+y$, $x+2y=$the series.

1. $2x^2-2xy+y^2=34$.

2. $2x^2+6xy+5y^2=130$.

3. Adding 1st and 2d, $4x^2+4xy+6y^2=164$.

4. Dividing 3d by 2, $2x^2+2xy+3y^2=82$.

5. Adding 1st and 4th, $4x^2+4y^2=116$.

6. Dividing, &c., $x=\sqrt{29-y^2}$

Substituting this value of x in the 1st, reducing, &c., $y=2$, and $x=5$.

The series is, 3, 5, 7, and 9.

28. $x=3$, $y=1$, and the number is 234.

29. Let $x=$2d term, and $y=$common difference.

Then $x-y$, x, $x+y$, $x+2y=$the series 2, 6, 10 and 14. Ans.

30. Let $2x=$common difference.

Since the sum of the series is 28, the sum of the means, or extremes, must be 14. Hence $7-x=$2d term.

And $7+x=$3d term.

Also $7-3x=$1st term.

And $7+3x=$4th term.

The series is, $7-3x$, $7-x$, $7+x$, $7+3x$.

Their product, $9x^4-490x^2+2401=585$.

Hence, $x=2$, and $2x=4$.

$$\left.\begin{array}{l}7-6,\ 7-2,\ 7+2,\ 7+6;\\ \text{Or }\ 1,\ 5,\ 9,\ 13.\end{array}\right\}\text{Ans}$$

SECTION XIV.

GEOMETRICAL PROPORTION AND PROGRESSION

ART. 357

1. See Book

2 3

3. 32 and 24.

4. 10 and 8.

5. 8 and 6.

6. $x : 20-x :: 3^2 : 1^2$.

Hence, 18 and 2 are the Nos.

By Art. 240, $\sqrt{18\times 2}=6$, the mean proportional.

7. 6 and 4.
8. 20 and 24.
9. He had 45 bushels, and sold 20.
10. 15 and 9.
11. 10 and 2.
12. 18 and 6.
13. 25 R. and 5 B.
14. 24 and 16.
15. 20 and 16.
16. 32 and 18.

ART. 371.

1. See Book.
2. $\frac{1}{3}$, 1 and 3.

ART. 373

3. See Book.
4. $\frac{121}{162}$.
5. 265720.
6. $\frac{174075}{59049}$.
7. 8192.
8. 531441.
9. \$9,265,100,944,259.20.

ART. 375

10. See Book.

11. Let x=the 1st term, and y=the ratio

Then the series is, x, xy, xy^2.

1. By the conditions, $x^3y^3=64$.
2. And $x^3+x^3y^3+x^3y^6=584$.
3. Extracting root of 1st, $xy=4$.

$$x=\frac{4}{y}.$$

4. Subtracting 1st from 2d, $x^3+x^3y^6=520$.
5. Substituting value of x, $\frac{64}{y^3}+64y^3=520$.
6. Completing square, &c., $y=2$, and $x=2$

The numbers required are, 2, 4 and 8

12. 2, 10 and 50.

13. Let the series be, x, xy, xy^2, xy^3.

1. Then by conditions, $x+xy=15$
2. $xy^2+xy^3=60$

3. Dividing the 2d by the 1st, $y^2=4$.
Then $y=2$, and $x=5$.
The numbers are 5, 10, 20 and 40.

14. Let x=last term.
Then $x+90$=first.
And $\sqrt{x^2+90x}$=second. (Art. 340.)
$2x+90+\sqrt{x^2+90x}=210.$
$x=30.$
The parts were 120, 60 and 30.

15. 5, 10 and 20.

16. Let x=the 1st term, and y=the ratio.
The series is, x, xy, xy^2, xy^3.
1. By conditions, $x+xy^3 : xy+xy^2 :: 7 : 3$.
2. Dividing by x, $1+y^3 : y+y^2 :: 7 : 3$.
3. By composition, (Art. 350. 2.)
$y^3+y^2+y+1 : y^2+y :: 10 : 3$.
4. Dividing by $y+1$, $y^2+1 : y :: 10 : 3$.
5. By Art. 337, $3y^2+3=10y$.
6 Completing square, &c., $y=3$.
And $x=1$.
The series is 1, 3, 9 and 27.

SECTION XV.

EVOLUTION OF COMPOUND QUANTITIES.

ART. 376.

1. See Book.
2. $a+2$.
3. $a+b$.
4. $a-2b$.
5. $2a-3b+4h$.

ART. 377.

6. See Book.
7. $1-2b+y$.
8. a^3-a^2+a.

9. a^2+2b-2.
10. x^2-2x+1.
11. x^2-2x+1
12. $2x^2-x+3$
13. $2a-3x$.
14. $x+1$.
15. $a-b$.

SECTION XVI.

APPLICATION OF ALGEBRA TO GEOMETRY

Art. 399.

Probs. 1 and 2. See Book.

Prob. 3. Let x=base.

Then $x-6$=perpendicular.

By Euc. 47. 1, $(30)^2=x^2+(x-6)^2$

Completing the square, &c., $x=24$ Ans

Prob. 4. Let $4x$=base.

Then will $3x$=the perpendicular.

Euc. 47. 1, $16x^2+9x^2=2500$.

By evolution, &c., $x=10$.

Ans. $4x=40$, the base; $3x=30$, the perpendicular

Prob. 5. $BC=6$, and $AB=8$.

Prob. 6. Let a=the given area.

And $DE=BF=b$. (Fig. 12.)

Also $EB=DF=d$, and $BC=x$.

1. By similar triangles $x-b:d::x:AB$.
2. Therefore $dx=(x-b)\times AB$.
3. By Art. 393, $a=AB\times\frac{1}{2}x$
4. Dividing by $\frac{1}{2}x$, $\frac{2a}{x}=AB$.
5. Substituting $\frac{2a}{x}$ for AB in 2d, $dx=2a-\frac{2ab}{x}$
6. Clearing of fractions, &c., $x=\frac{a}{d}\pm\sqrt{\frac{a^2}{d^2}-\frac{2ab}{d}}=BC$

Prob. 7. Let b=base, p=perpendicular, h=hypothenuse of the given triangle, d=BD the perpendicular drawn, and x=the greater segment. (Fig. 13.)

1. By Euc. 47. 1, $x^2+d^2=b^2$.
2. " " " $(h-x)^2+d^2=p^2$.
3. By transposition in 1st, $d^2=b^2-x^2$.
4. " " 2d, $d^2=p^2-(h-x)^2$.
5. By comparison, (Art. 277,) $b^2-x^2=p^2-(h-x)^2$.
6. Reducing, &c., $x=\dfrac{b^2+h^2-p^2}{2h}$=AD.

Prob. 8. Draw CI perpendicular to AB. By supposition, DG is parallel to AB. Therefore,

The triangle CHG, is similar to CIB.
And CDG, to CAB.
Let CI=d DG=x.
AB=b The given area=a.

1. By similar triangles, CB : CG : : AB : DG
2. And CB : CG : : CI : CH.
3. By equal ratios, (Art. 346,) AB : DG : : CI : CH
4 Therefore, $\dfrac{\text{DG}\times\text{CI}}{\text{AB}}=\text{CH}$
5. By the figure, CI−CH=IH=DE.
6. Substituting for CH, $\text{CI}-\dfrac{\text{DG}\times\text{CI}}{\text{AB}}=\text{DE}$.
7. That is, $d-\dfrac{dx}{b}=\text{DE}$.
8. By Art. 388, $a=\text{DG}\times\text{DE}=x\times\left(d-\dfrac{dx}{b}\right)$.
9 That is, $a=dx-\dfrac{dx^2}{b}$.
10. This reduced, gives $x=\dfrac{b}{2}\pm\sqrt{\dfrac{b^2}{4}-\dfrac{ab}{d}}=\text{DG}$.

The side DE is found by dividing the area by DG.

Prob. 9. In the circle AQBR, (Fig. 15,) let P be a given point in the diameter AB. Also, let $AP=a$, $BP=b$, the given difference$=d$, and $PR=x$.

Then will $PQ=x+d$.

1. By Euc. 35. 3, $PR\times QR=AP\times BP$.
2. That is, $x\times(x+d)=a\times b$.
3. Or, $x^2+dx=ab$.
4. Completing square, &c., $x=-\frac{1}{2}d\pm\sqrt{\frac{1}{4}d^2+ab}=PR$.

Prob. 10. (Fig. 7.) Let $y=BG$, the greater of the given sides.

Then will $1155-y=AG$, the less side

And $x=AH$, the less segment.

And $x+495=BH$, the greater segment.

1. By Euc. 47. 1, $y^2=x^2+990x+245025+90000$.
2. By " " $y^2-2310y+1334025=x^2+90000$.
3. Subtr. 2d from 1st, $2310y-1334025=990x+245025$
4. Transposing, dividing, &c., $y=\dfrac{3x+4785}{7}$.
5. Substituting value of y in 1st, clearing of fractions, transposing, completing square, &c., $x=225$, the less segment.

And $225+495=720$, greater segment.

The sides are 945, 375 and 780. Ans.

Prob. 11. Let $x=$base AB, (Fig. 13,) $y=$the perpendic. BC

And $720-(x+y)=$the hypothenuse AC.

1. By similar triangles $720-(x+y):x::y:144$.
2. $xy=103680-144(x+y)$.
3. Euc. 47. 1, $x^2+y^2=(720-x-y)^2$.
4. Expand 3d, $x^2+y^2=518400-1440(x+y)+x^2+2xy+y^2$.
5. Transposing, &c., $259200+xy=720(x+y)$.
6. Subtr. 2d from 5th, transposing, &c.,

   $x+y=420$, and $x=420-y$.
7. Substituting value of x in 3d, $y^2+(420-y)^2=90000$.
8. Expanding, completing the square, &c., $y=240$, the perpendicular.

The sides are 300, 240 and 180

Prob. 12. Let x=side of square, and $x+d$=diagonal.

1. By Euc. 47. 1, $x^2+x^2=x^2+2dx+d^2$
2. Transposing, completing square, &c., $x=d\pm d\sqrt{2}$

Prob. 13. Let x=side of square, b=base, and h=height of the triangle.

By similar triangles $x : h-x :: b : h$.

Then, $hx=bh-bx$, and $x=\frac{bh}{b+h}$. Ans.

Prob. 14. Let x=base BC, y=BD, a=BA, c=AC and b=the bisecting line.

1 By Euc. 3. 6, $a : c :: y : (\text{DC})=\frac{cy}{a}$.
2. By Art. 350. 2, $a+c : a :: x : y$.
3. By Art. 337, $(a+c)y=ax$, and $y=\frac{ax}{a+c}$.
4. Substitut. the val of y in 1st, $a : c :: \frac{ax}{a+c} : \frac{acx}{a^2+ac}=\frac{cx}{a+c}$
5 Then $\frac{cx}{a+c}=\text{DC}$.
6. By Euc. B. 6, $ac=\frac{acx^2}{(a+c)^2}+b^2$
7. Transposing, &c., $acx^2=(ac-b^2)\times(a+c)^2$
8. Removing co-efficient, &c., $x=a+c\times\sqrt{\frac{ac-b^2}{ac}}$

Prob. 15. Let x=perpendicular, and y=base of the triangle.

By similar triangles, $x : y :: x-12 : 12$

1. By Art. 337, $12x=xy-12y$.
2. Transposing and multiplying by 2, $2xy=24(x+y)$.
3. By Euc. 47. 1, $x^2+y^2=1225$.
4. Adding 2d and 3d, $x^2+2xy+y^2=1225+24(x+y)$.
5. Substituting z for $x+y$, $z^2=1225+24z$.
6. Completing square, &c., $z=49$, that is, $x+y=49$
7. Then $x=49-y$.
8. Substituting the value of x in 3d, &c., $y^2-49y=-588$.
9. Completing square, &c., $\left.\begin{array}{r} y=28. \\ x=21. \end{array}\right\}$ Ans

Prob. 16. Let $4x$=base, then $3x$=perpendicular of triangle.

1. By Art. 393, $6x^2$=area.
2. By the conditions, $6x^2-7x$=hypothenuse.
3. By Euc. 47. 1, $16x^2+9x^2=36x^4-84x^3+49x^2$.
4. Dividing by x^2, $16+9=36x^2-84x+49$.
5. By evolution, $5=6x-7$; and $x=2$.

$4x=8$ the base, 6 the perpendicular, and 10 the hypothenuse.

Prob. 17. Let x=the width of the walk in rods.

Then $36x+4x^2$= areas of the two walks on the sides.

And $24x$= " " " ends.

By the conditions, $4x^2+60x=216$.

Reducing the equation, $x=3$ rods. Ans.

Prob. 18. Let $6x$=length, and $5x$=breadth.

By the conditions, $30x^2\div 6=125$.

Ans, 30 and 25.

Prob. 19. Let x=diagonal of parallelogram, and consequently the perpendicular of triangle.

By Art. 393, $60x\div 2=30x$=area of triangle.

By the conditions, $5:8::30x:48x$=area of rectangle.

As one side of the rectangle is 60,

The other, (Art. 395,) will be $=\frac{48x}{60}=\frac{4x}{5}$.

By Euc. 47. 1, $\frac{16x^2}{25}+3600=x^2$.

Reducing the equation, $x=100$, the diagonal

$$\left.\begin{array}{l}4800\text{=area of rectangle.}\\ 3000\text{=area of triangle.}\end{array}\right\}\text{Ans.}$$

Prob. 20. Let x=side of the less; and y=side of the greater.

1. Then by the conditions, $x^2y+20=xy^2$.
2. $x^2y:xy^2::4:5$.
3. $5x^2y=4xy^2$.
4. Dividing by xy, $5x=4y$.
5. Removing co-efficient, $x=\frac{4y}{5}$.

6. Substituting the value of x in 1st, $\frac{16y^3}{25}+20=\frac{4y^3}{5}$.

7. Clearing of fractions, &c., $y=5$, and $x=4$.
The depth of the less$=5$, the greater$=4$.

Prob. 21. Draw an equilateral triangle, and from one of the angles F, let fall the perpendicular FC to the opposite side, bisecting it in C. (Euc. 10. 1.)

Let $2x$=one side of the triangle, and y=the perpendicular let fall, and a, b and c=the 3 perpendiculars.

By Euc. 47. 1, $4x^2-x^2=y^2$.

By evolution, &c., $y=x\sqrt{3}$.

Art. 393, xy=area of trian.$=x^2\sqrt{3}$ by substi. val. of y.

And $ax+bx+cx$=the area of triangle. (Art. 393.)

Then $ax+bx+cx=x^2\sqrt{3}$.

Dividing by x, &c., $x=\frac{a+b+c}{\sqrt{3}}$. Ans.

Prob. 22. Let x=breadth of street, then $9x-3$=side of square.

Then will $22x^2-6x$=area of two longest streets.

And $18x^2-6x$=area of two shortest streets.

By the conditions, $40x^2-12x=36x-12+228$.

Then 24=side of square, and 576=area of square.

Prob. 23. Draw the right-angled triangle ABC; from the acute angle A, draw the line AD bisecting the opposite side CB in D. And from the other acute angle C, draw the line CE, bisecting the opposite side AB in E.

Let x=half the base, a=AD, y=half the perpendicular, and b=CE.

1. By Euc. 47. 1, $4y^2+x^2=a^2$.
2. By " " $y^2+4x^2=b^2$.
3. By transposition, $y^2=b^2-4x^2$.
4. Substituting value of y^2 in 1st, $4b^2-16x^2+x^2=a^2$
5. Trans., &c., $x=\sqrt{\frac{4b^2-a^2}{15}}$; and $y=\sqrt{\frac{4a^2-b^2}{15}}$.

MISCELLANEOUS PROBLEMS.

1. 9 and 19.
2. 17 and 31.
3. 13.
4. 84.
5. £180.
6. 864.
7. 21 and 16.
8. 3 and 9.
9. 12 hours.
10. 6 and 9.
11. Worked 156 days, and was idle 234.
12. 53.
13. 33 lbs. at 13*s*. 6*d*., and 71 lbs. at 9*s*. 6*d*.
14. 23.
15. $\frac{5}{12}$.
16. 126 gallons.
17. 13 and 20.
18. 233 and 142 votes.
19. 24 feet.
20. $120.
21. 16 years.
22. 54 years.
23. 21 years.
24. 6$\frac{3}{4}$.
25. 56 hours.
26. 20 days.
27. 197 sheep.
28. 6 days.
29. 105 days.
30. 30 days.
31. 2 hours
32. 3 pence and 36 farthings.
33. 5 shares; 10*s*. distributed.
34. 300 leaps.
35. One 7, the other 5 minutes.
36. 60 days. A will go 1800, and B 1200 miles.
37. £88 from A; £44 from B.
38. A $312, B $412, C $476.
39. First $12, second $2, and third $16.
40. 5 hours and 30 minutes.
41. 2 dollars.
42. 13 hours and 24 minutes.
43. The 2d would fill it in 32 hours, and the 3d would empty it in 19$\frac{1}{5}$ hours.
44. $26300.
45. 1 hour and 5$\frac{5}{11}$ minutes.
46. Wheat 3*s*., and corn 2*s*.
47. Men $5, and boys 2\frac{1}{2}$.
48. 5, 15, 2 and 50.
49. A 14$\frac{34}{49}$, B 17$\frac{23}{41}$, C 23$\frac{7}{31}$.
50. A 26, B 14, and C 8.
51. 9 and 6.
52. The side is 44 rods.
53. 24 and 18 years.
54. 29 years.
55. 36 rows, 25 trees in a row.
56. 92160 acres.
57. 20 lbs. for 8 crowns.
58. 3, 7, 11 and 15.
59. 2, 4 and 8.
60. 5 and 12.
61. 945, 375 and 780 feet.
62. Sides 30 and 40; P 24.

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